

**ELECTROWEAK CONSTRAINTS FROM  
ATOMIC PARITY VIOLATION AND NEUTRINO SCATTERING***Timothy Hobbs*

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**Abstract**

Precision electroweak physics can provide fertile ground for uncovering new physics beyond the Standard Model (SM). One area in which new physics can appear is in so-called “oblique corrections,” *i.e.*, next-to-leading order expansions of bosonic propagators corresponding to vacuum polarization. One may parametrize their effects in terms of quantities  $S$  and  $T$  that discriminate between conservation and non-conservation of isospin. This provides a means of comparing the relative contributions of precision electroweak experiments to constraints on new physics. Given the prevalence of strongly  $T$ -sensitive experiments, there is an acute need for further constraints on  $S$ , such as provided by atomic parity-violating experiments on heavy atoms. We evaluate constraints on  $S$  arising from recently improved calculations in the Cs atom. We show that the top quark mass  $m_t$  provides stringent constraints on  $S$  within the context of the SM. We also consider the potential contributions of next-generation neutrino scattering experiments to improved  $(S, T)$  constraints.

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**I. INTRODUCTION**

The search for exotic physics not explicable in terms of the standard  $SU(3) \times SU(2) \times U(1)$  model has been carried out on the complementary frontiers of energy and precision. Exploration on the precision frontier has included a wide range of measurement in electroweak physics, seeking to reduce experimental uncertainties of common inputs and other observables of the SM. Present and proposed versions of such experiments promise unprecedented measurements of such basic SM quantities as the electroweak mixing angle, providing needed tests of the SM and searches for Beyond-Standard-Model (BSM) physics.

A formalism that broadly incorporates the effects of hypothetical new physics can be achieved through the introduction of vacuum polarization loop corrections to the vector boson propagators of the electroweak theory; an example of this modification is illustrated in Fig. 1 for the leading-order (LO) photon propagator in QED. Such corrections modify the LO theory in a way whereby effects of exotic physics outside the SM can be represented in terms of a small number of “oblique corrections” (OCs) affecting gauge boson propagators.

These are to be distinguished from more direct, one-loop corrections at the level of the vertex in the electroweak theory.

It is possible to express obliquely-corrected quantities in terms of parameters  $S$  and  $T$  [1], permitting one to compare the relative contributions of various precision electroweak experiments to constraints on new physics. Several experiments constrain  $T$  well, but there is a scarcity of constraints on  $S$ , such as those provided by atomic parity-violating experiments on heavy atoms [2].

We briefly review the  $(S, T)$  formalism in Sec. II. We then evaluate in Sec. III constraints on  $S$  arising from recently improved calculations in the Cs atom. We also notice that within the Standard Model, the top quark mass  $m_t$  provides stringent constraints on  $S$  (Sec. IV). In Sec. V we consider the potential of next-generation neutrino scattering experiments for improving  $(S, T)$  constraints. We conclude in Sec. VI.

## II. OBLIQUE CORRECTIONS

The tools for approximating the effects of loop corrections in QCD are best understood via analogy with the simpler  $U(1)$  model of QED. In this setting, the addition of a single loop correction associated with vacuum polarization also modifies the photon propagator. So, the exchange of a photon with 4-momentum transfer  $q^2$  is modified according to

$$\frac{-ig_{\mu\nu}}{q^2} \rightarrow \frac{-ig_{\mu\nu}}{q^2} + \left(\frac{-ig_{\mu\alpha}}{q^2}\right)\Pi^{\alpha\beta}\left(\frac{-ig_{\beta\nu}}{q^2}\right), \quad (1)$$

where the amplitude of the loop insertion is calculable according to the Feynman diagram shown in Fig. 1, viz.:

$$\Pi^{\alpha\beta}(q) = (-1) \int \frac{d^4k}{(2\pi)^4} \text{Tr}[(ie\gamma^\alpha) \frac{i(\not{k} + m)}{k^2 - m^2} (ie\gamma^\beta) \frac{i(\not{q} + \not{k} + m)}{(q + k)^2 - m^2}] = (q^2 g^{\alpha\beta} - q^\alpha q^\beta) \Pi(q^2). \quad (2)$$

As the integral in the loop amplitude is logarithmically divergent, it may be renormalized by imposing a scale cutoff  $\Lambda$  as

$$\Pi(q^2) \approx \frac{e_0^2}{12\pi^2} \ln\left(\frac{\Lambda^2}{q^2}\right). \quad (3)$$

Thus, in this case, higher-order OCs may be recast as a modification of the amplitude of the virtual photon exchange as an infinite power series of logarithmically divergent integrals, with successive terms suppressed by higher powers of  $\alpha_0$ . Standard inputs of the  $U(1)$  theory are then modified as

$$\alpha(q^2) = \alpha_0(1 - \dots \pm (\alpha_0 \Pi(q^2))^n) = \frac{\alpha_0}{1 + \alpha_0 \Pi(q^2)}. \quad (4)$$

In the transition to the electroweak theory, the essential difference from the foregoing QED ansatz for incorporating higher-order OCs is the expanded dimensionality of the group of electroweak generators. In an analogous fashion, the propagators of the vector bosons  $Z$  and  $W$  may also be expanded in terms of loop amplitudes proportional to logarithmically divergent integrals. These amplitudes must be indexed over the  $SU(2) \otimes U(1)$  group generators. A particularly convenient means of collecting the leading-order contributions of the

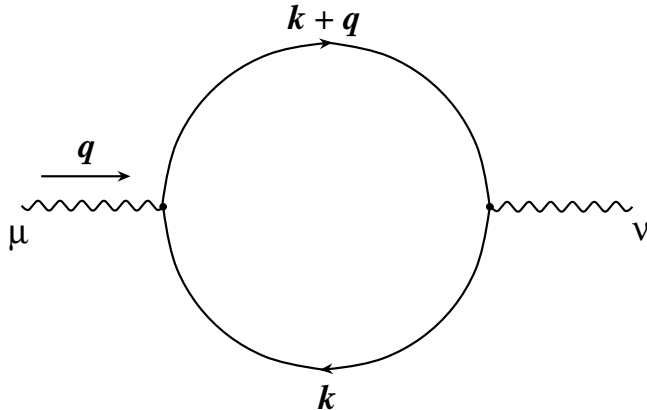


FIG. 1: The order- $\alpha$  correction to the photon propagator of QED. This diagram corresponds to the amplitude to spontaneously generate an electron-positron pair from the vacuum; the mathematical details of this process are similar in character to the self-energy corrections to electroweak propagators.

indexed loop amplitudes is to write the parameters [1]

$$S = 16\pi \frac{d}{dq^2} [\Pi_{33}(q^2) - \Pi_{3Q}(q^2)]_{q^2=0} \quad (5a)$$

and

$$T = \frac{4\pi}{\sin^2 \theta_W \cos^2 \theta_W m_Z^2} [\Pi_{11}(q^2) - \Pi_{33}(q^2)]_{q^2=0}. \quad (5b)$$

These parameters are suitable for computing and constraining OCs in that, for the case of virtual  $Z$ -boson exchange, the modifications of the relevant standard model inputs are expressible in terms of linear combinations of  $S, T$ . (We neglect here a parameter  $U$  which arises only in particular models and leads to a difference in  $S$  when applied to  $Z$  and  $W$  propagators.) One can thus represent SM observables for small deviations from nominal values as  $\bar{x} = \bar{x}_0 + AS + BT$ , where  $A, B$  are constants;  $\bar{x}$  is a measured quantity; and  $\bar{x}_0$  is its value for nominal parameters corresponding to  $S = T = 0$ . Such relations imply that measured values of SM inputs, with associated experimental uncertainties, correspond to unique linear bands in the  $(S, T)$  plane. The combination of bands with different slopes constrains the space of OCs to an elliptical region. In this way we may test the effects on the allowed  $(S, T)$  region of alterations in any experimental inputs. For nominal inputs we take values quoted in Ref. [3], suitably updated with more recent calculations when available. We use an effective value of the weak mixing angle as measured via leptonic vector and axial-vector couplings:  $\sin^2 \theta^{\text{eff}} \equiv (1/4)(1 - [g_V^l/g_A^l])$ .

### III. CONSTRAINTS FROM $Q_W(\text{Cs})$

The orientation of the ellipse of OCs [3] suggests an acute need for additional constraints on  $S$ . Measurements of  $Q_W$  in atomic parity violation experiments promise special sensitivity to  $S$  with almost none to  $T$  [2, 4]. For this reason, there has been great interest in improved measurements of  $Q_W$  for heavy nuclei. Aside from atomic parity violation, the electroweak

TABLE I: Electroweak observables leading to  $S, T$  constraints.

| Quantity                    | Experimental<br>value | Theoretical<br>value            |
|-----------------------------|-----------------------|---------------------------------|
| $Q_W(\text{Cs})$            | $-73.20 \pm 0.35$     | $-73.15 - 0.800S - 0.007T$      |
| $M_W(\text{GeV}/c^2)$       | $80.399 \pm 0.023$    | $80.385 - 0.29S + 0.45T$        |
| $g_L^2(\nu\text{NC})$ [12]  | $0.3027 \pm 0.0018^a$ | $0.3041 - 0.00272S + 0.00665T$  |
| $g_R^2(\nu\text{NC})$ [12]  | $0.03076 \pm 0.00110$ | $0.0300 + 0.00094S - 0.00020T$  |
| $\Gamma_{ll}(Z)$ (MeV)      | $83.984^b \pm 0.087$  | $84.005^b - 0.18S + 0.78T$      |
| $\sin^2\theta^{\text{eff}}$ | $0.23149 \pm 0.00016$ | $0.23140 + 0.00361S - 0.00257T$ |

<sup>a</sup> Modification by [11] of value  $g_L^2 = 0.30005 \pm 0.00137$  quoted in [13]

<sup>b</sup> Values quoted in Ref. [11]

observables with the greatest ratio of  $S$  to  $T$  coefficients (i.e.,  $A/B$ ) are the cross section for  $\bar{\nu}_e \rightarrow \bar{\nu}_e$ , whose potential impact has been discussed in Ref. [5], as well as the top quark mass  $m_t$ , whose effect within the Standard Model is noted below in Sec. IV.

The combination of measurements and theory provides the strongest constraints at present for atomic cesium ( $Z = 55$ ) [6, 7], particularly in view of the latest theoretical atomic physics calculations [8]. We therefore restrict our attention to cesium in what follows. For a cesium isotope of  $N$  neutrons, the inclusion of next-to-leading-order loop corrections yields [2]

$$Q_W(^{55+N}\text{Cs}) = (0.9857 \pm 0.0004)\rho[-N + 55(1 - (4.012 \pm 0.01)\sin^2\theta_W)] . \quad (6)$$

If one assumes that custodial symmetry (corresponding to  $\rho = 1$ ) is broken, the previous expression may be expanded using  $\rho = 1 + \alpha T$  and the standard, linearized expression  $\sin^2\theta^{\text{eff}} = 0.2314 + 0.00361S - 0.00257T$  [3]:

$$Q_W(^{133}\text{Cs}) = -73.16 - 0.800S - 0.007T , \quad (7)$$

in which we have taken the stable isotope with  $N = 78$  for Cs. A small change from the central value of  $-73.19$  quoted in Ref. [3] is due to corrections detailed in Ref. [9, 10]. Clearly, incremental reductions to the uncertainty of  $Q_W$  will necessarily constrain  $S$  dramatically more tightly than  $T$ . Updated calculations [9, 11] incorporating the atomic physics corrections of Ref. [8] and the two experimental measurements [6, 7] yield

$$Q_W(^{133}\text{Cs}) = -73.20(35) . \quad (8)$$

Neglecting the  $T$  dependence in Eq. (7), this implies a value  $S = 0.06 \pm 0.44$ . The error on  $S$  has been reduced by more than a factor of 2 since the analysis in Ref. [3]. We now discuss the impact of this measurement in light of other electroweak constraints.

We use the same constraints as in an earlier analysis [3], suitably updated to reflect an erratum in Ref. [12], further corrections to the NuTeV result detailed in Ref. [11], and the latest averages of  $M_W$  and  $\sin^2\theta^{\text{eff}}$  [13]. We omit an older constraint with large error bars based on parity violation in atomic thallium. The inputs are summarized in Table I.

Constraints on  $(S, T)$  due to the inputs in Table I are summarized in Fig. 2. The (solid, dashed) ellipses correspond to constraints (with, without) the atomic parity violation measurement on the first line of Table I. The constraints due to  $M_W$  (imposed) and  $m_t$  (not

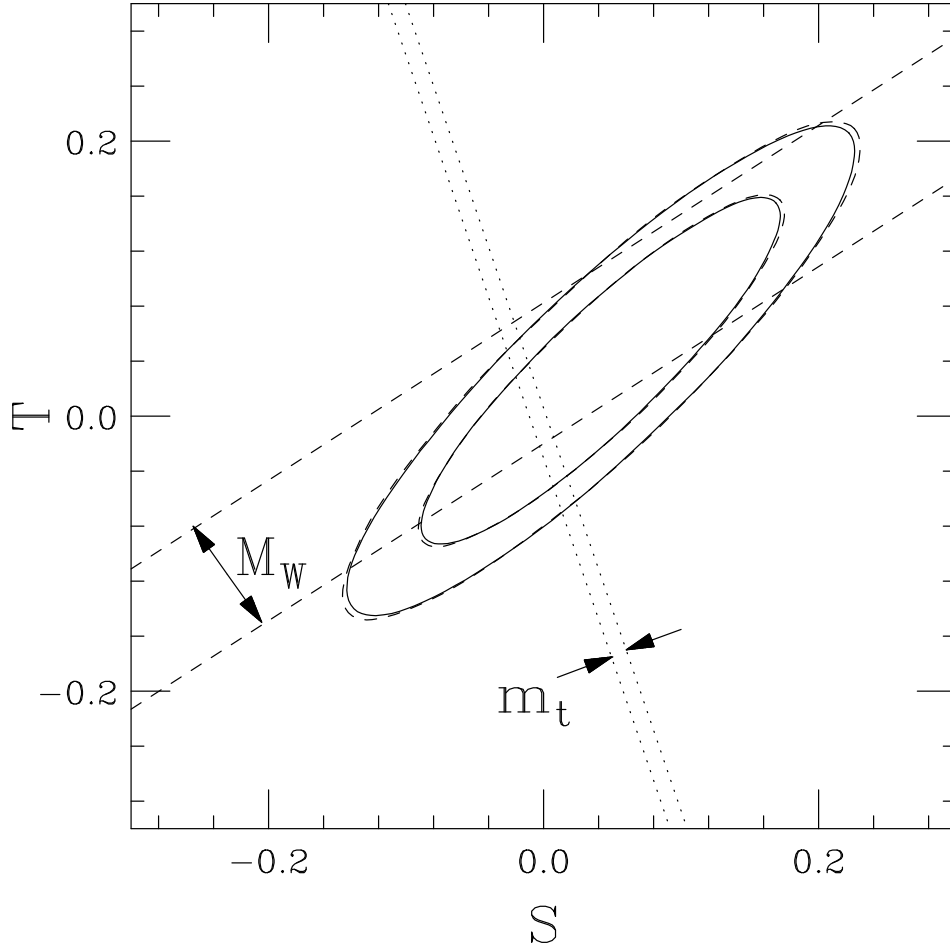


FIG. 2: Effects of atomic parity constraints in atomic Cs [6–8] on OCs in  $S, T$  space. Inner and outer ellipses denote 68% and 90% confidence level (c.l.) limits. Solid and dashed curves are plotted with and without atomic parity constraints (first line of Table I). Dashed and dotted bands denote constraints due to  $M_W$  (imposed) and  $m_t$  (not imposed here but discussed below).

imposed here, but to be discussed subsequently) are illustrated by the dashed and dotted bands, respectively.

Comparing the dashed and solid ellipses in Fig. 2, one sees that the imposition of the constraints due to atomic parity violation has a small effect on the allowed  $(S, T)$  region. Measurements at LEP and the Fermilab Tevatron have provided sufficiently tight constraints that the value of  $S = 0.06 \pm 0.44$  from parity violation in atomic cesium is overshadowed by the (approximate) constraint  $|S| < 0.16$  due to the remaining observables. Thus, a further reduction in errors by a factor of at least three would be required for atomic parity violation to significantly affect the constraints on  $(S, T)$ .

#### IV. TOP QUARK MASS CONSTRAINTS

Instead of  $(S, T)$  one often employs the measured quantities  $(m_t, m_W)$  to express constraints of electroweak measurements. Using standard expressions to approximate the loga-

rithmic divergences of equations (5a, 5b),  $S$  and  $T$  may be written [2]

$$S \approx \frac{1}{6\pi} \ln \frac{m_H}{\Lambda_H} , \quad (9)$$

$$T \approx \frac{3}{16\pi \sin^2 \theta_W} \left[ \frac{m_t^2 - \Lambda_t^2}{m_W^2} \right] - \frac{3}{8\pi \cos^2 \theta} \ln \frac{m_H}{\Lambda_H} . \quad (10)$$

Here we take the nominal parameters corresponding to  $S = T = 0$  to be  $\Lambda_t = 174$  GeV (as in Ref. [3]) and  $\Lambda_H = 100$  GeV. We may then write

$$T = \frac{3(m_t^2 - \Lambda_t^2)}{16\pi \sin^2 \theta_W m_W^2} - \frac{9S}{4 \cos^2 \theta_W} , \quad (11)$$

yielding a linear relation for the SM observable  $m_t^2$  in terms of  $S, T$ :

$$m_t^2 = m_{W,0}^2 \left[ 12\pi \tan^2 \theta_W S + \frac{16\pi \sin^2 \theta_W}{3} T \right] + \Lambda_t^2 , \quad (12)$$

in which  $m_{W,0}$  is the central value of the  $W$  mass. We may linearize this expression in  $m_t$  about the value  $\Lambda_t = 174$  GeV, finding

$$m_t = (174 + 210.8S + 72.0T) \text{ GeV}/c^2 . \quad (13)$$

The effect of including the observed top quark mass (Tevatron average [14]),

$$m_t = 173.1 \pm 1.3 \text{ GeV}/c^2 , \quad (14)$$

is shown in Fig. 3. Although it would be inappropriate to exploit Eq. (13) to derive a new constraint in parameter space for  $S$  and  $T$  very far from zero, we use it only for  $S$  and  $T$  very near zero, consistent with Eqs. (9) and (10), respecting the constraints provided by  $m_t$  and  $m_W$ .

The linear relation (13), when combined with the experimental value (14), leads to a very narrow allowed band in  $(S, T)$  space which is almost perpendicular to the semi-major ellipse in Fig. 2, thus acting to drastically reduce the allowed  $(S, T)$  region. One may then ask what experiments are likely to provide the most incisive further constraints on  $S$  and  $T$ . The answer is those that provide bands nearly perpendicular to the semi-major ellipse in Fig. 3. For an observable  $\bar{x}$  whose  $(S, T)$  dependence is expressed as  $\bar{x} = \bar{x}_0 + AS + BT$ , such an observable would have

$$210.8A + 72.0B = 0 \quad , \quad \text{or} \quad B/A = -2.93 \quad . \quad (15)$$

These ratios for the observables  $(M_W, g_L^2, g_R^2, \Gamma_u(Z), \sin^2 \theta^{\text{eff}})$  listed in Table I are  $(-1.55, -2.44, 0.21, -4.33, -0.71)$ , respectively. Of these, the observable  $g_L^2(\nu\text{NC})$  has the  $B/A$  ratio closest to that in (15). A proposal to improve the measurement of this observable (along with several others connected with neutrino scattering) has been made in Ref. [15], which we now discuss.

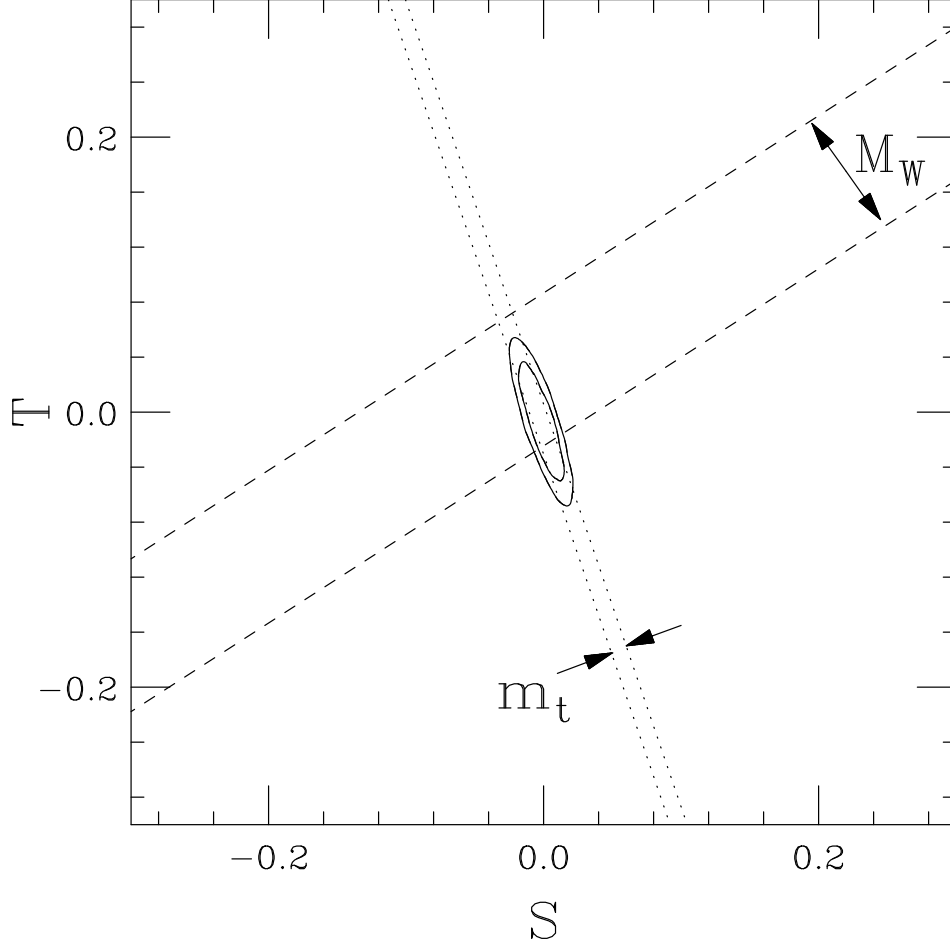


FIG. 3: Effects of including top quark mass constraint  $m_t = 173.1 \pm 1.3 \text{ GeV}/c^2$  on constraints in  $S, T$  space. Inner and outer ellipses denote 68% and 90% confidence level (c.l.) limits. Solid and dashed curves are plotted with and without atomic parity constraints. Dashed and dotted bands denote constraints due to  $M_W$  and  $m_t$ , respectively.

## V. CONSTRAINTS FROM NEUTRINO SCATTERING

For several decades, neutrino deep inelastic scattering ( $\nu$ DIS) has shed tremendous light on the electroweak interactions, through the measurement of the ratio of neutral-current and charged-current events. The quantities  $g_L^2$  and  $g_R^2$  in Table I reflect the contribution to  $S, T$  constraints of the most recent  $\nu$ DIS experiment performed by the NuTeV Collaboration [12]. These coupling constants may be expressed as

$$g_L^2 = (2g_L^\nu g_L^u)^2 + (2g_L^\nu g_L^d)^2 = \rho^2 \left( \frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W \right) \quad (16a)$$

and

$$g_R^2 = (2g_L^\nu g_R^u)^2 + (2g_L^\nu g_R^d)^2 = \rho^2 \left( \frac{5}{9} \sin^4 \theta_W \right). \quad (16b)$$

Here we have used the parameter  $\rho = 1 + \alpha T$ , where the fine structure constant  $\alpha$  is taken to have its value of  $\sim 1/128 = 0.0078$  at the electroweak scale. Linearizing these equations

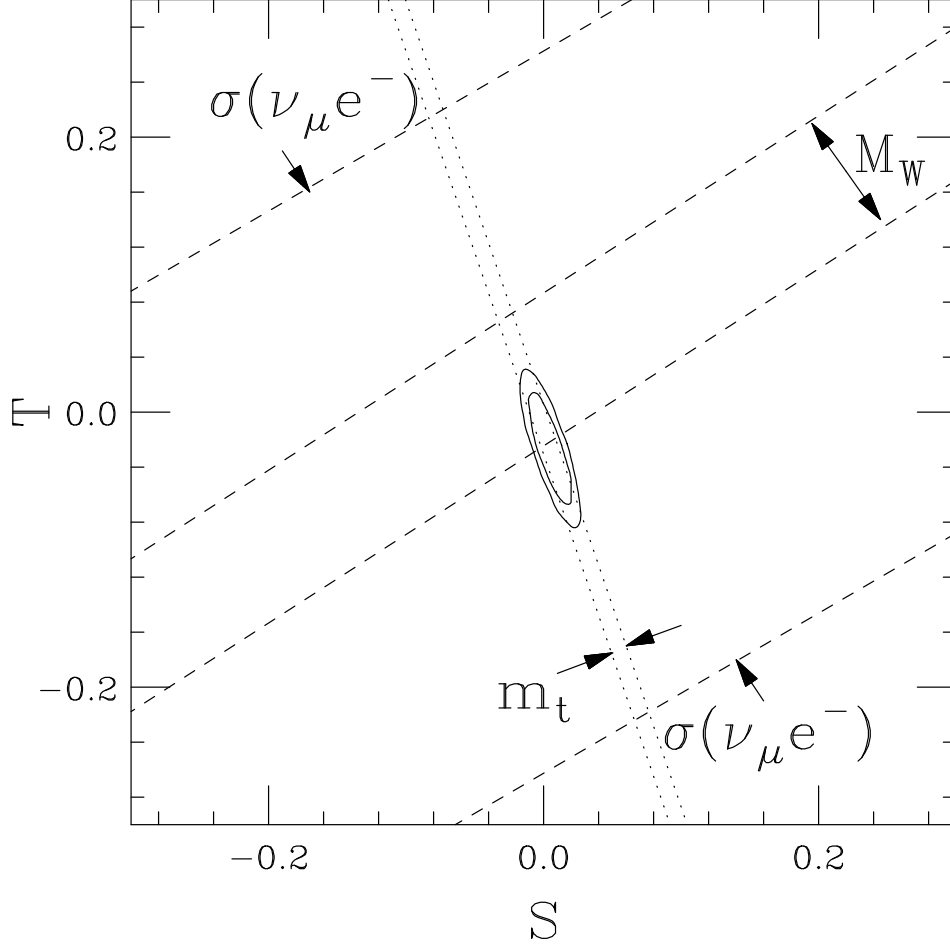


FIG. 4: Effects of including top quark mass constraint  $m_t = 173.1 \pm 1.3 \text{ GeV}/c^2$  on constraints in  $S, T$  space. The same data have been used as in Fig. 3 except that errors on  $g_L^2$  and  $g_R^2$  have been reduced by a factor of 3, and constraints due to  $\nu_\mu e^-$  elastic scattering (ES) (broad dashed band) have been imposed. Inner and outer ellipses denote 68% and 90% confidence level (c.l.) limits. Narrow dashed and dotted bands denote constraints due to  $M_W$  and  $m_t$ , respectively.

in  $S$  and  $T$ , one obtains the  $S$  and  $T$  dependence noted in Table I.

The NuSOnG Collaboration has recently proposed remeasuring  $g_L^2$  and  $g_R^2$  [15] with approximately a factor of two greater precision. Assuming such errors and maintaining the same central values as reported by NuTeV [12], we find hardly any effect on the allowed  $S, T$  ellipse. Assuming instead the errors on these quantities are divided by a factor of 3, the result is shown in Fig. 4. In this Figure we have also included the effects of measuring  $\sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-)$  to 0.7% as advocated in Ref. [15]. The inputs to the fit are summarized in Table II.

The  $B/A$  ratio for  $\nu_\mu e^-$  ES is  $-1.72$ , which would be quite favorable for constraining the  $(S, T)$  error ellipse if the uncertainty were about 1/4 the value anticipated in Ref. [15].



TABLE II: Electroweak observables leading to  $S, T$  constraints, including top quark mass and possible improvements in neutrino scattering measurements.

| Quantity                                | Experimental<br>value | Theoretical<br>value            |
|---|-----------------------|---------------------------------|
| $Q_W(\text{Cs})$                        | $-73.20 \pm 0.35$     | $-73.15 - 0.800S - 0.007T$      |
| $M_W(\text{GeV}/c^2)$                   | $80.399 \pm 0.023$    | $80.385 - 0.29S + 0.45T$        |
| $g_L^2(\nu\text{NC})$ [12] <sup>a</sup> | $0.3027 \pm 0.0006$   | $0.3041 - 0.00272S + 0.00665T$  |
| $g_R^2(\nu\text{NC})$ [12] <sup>a</sup> | $0.03076 \pm 0.0004$  | $0.0300 + 0.00094S - 0.00020T$  |
| $\Gamma_{ll}(Z)$ (MeV)                  | $83.984 \pm 0.087$    | $84.005 - 0.18S + 0.78T$        |
| $\sin^2\theta^{\text{eff}}$             | $0.23149 \pm 0.00016$ | $0.23140 + 0.00361S - 0.00257T$ |
| $m_t$                                   | $173.1 \pm 1.3$       | $174 + 210.8S + 72.0T$          |
| $\sigma(\nu_\mu)e^-$ <sup>b</sup>       | $0.356 \pm 0.0025$    | $0.356 - 0.00554S + 0.00952T$   |

<sup>a</sup> Present errors assumed to be divided by 3.

<sup>b</sup> Anticipated [15]; units of  $G_F^2 m_e E_\nu / (2\pi)$ .

## VI. CONCLUSIONS AND OUTLOOK

The  $(S, T)$  language is a broad signal for BSM physics. A complete set of observables of the SM are expressible to first order in terms of  $S, T$  and, resultantly, separate measurements of SM observables may independently and uniquely constrain the space of OCs. It then is possible to quickly evaluate the impact of any improvements in present experiments or totally new measurements.

In spite of the sensitivity of atomic parity violation measurements to  $S$ , we have seen that order-of-magnitude reductions to  $Q_W$  are likely necessary (compared to the  $\sim 50\%$  reductions that represent improvements in the past eight years) before such experiments significantly constrain  $S$ .

Similarly ambitious undertakings in neutrino physics will be needed in order to provide significant improvements in  $(S, T)$  constraints. Both measurements of  $\nu\text{DIS}$  and  $\nu_\mu e^-$  ES, foreseen in Ref. [15], will have to represent ambitious, order-of-magnitude improvements over current experiments in order to have a significant impact on the  $(S, T)$  plane.

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